

Consequences of nonconformist behaviors in a continuous opinion model

Allan R. Vieira¹, Celia Anteneodo^{2,3}, Nuno Crokidakis¹

¹ Instituto de Física, Universidade Federal Fluminense, Niterói/RJ, Brazil

² Departamento de Física, PUC-Rio, Rio de Janeiro/RJ, Brazil

³ National Institute of Science and Technology for Complex Systems, Brazil

E-mail: allanrv@if.uff.br, celia.fis@puc-rio.br, nuno@if.uff.br

Abstract. We investigate opinion formation in a kinetic exchange opinion model, where opinions are represented by numbers in the real interval $[-1, 1]$ and agents are typified by the individual degree of conviction about the opinion that they support. Opinions evolve through pairwise interactions governed by competitive positive and negative couplings, that promote imitation and dissent, respectively. The model contemplates also another type of nonconformity such that agents can occasionally choose their opinions independently of the interactions with other agents. The steady states of the model as a function of the parameters that describe conviction, dissent and independence are analyzed, with particular emphasis on the emergence of extreme opinions. Then, we characterize the possible ordered and disordered phases and the occurrence or suppression of phase transitions that arise spontaneously due to the disorder introduced by the heterogeneity of the agents and/or their interactions.

Keywords: Dynamics of social systems, Collective phenomena, Computer simulations, Critical phenomena

1. Introduction

Statistical physics furnishes diverse tools for the study of human social dynamics. Despite the apparent dissimilarities between social and physical systems, many social processes present a phenomenology that resembles that found, for instance, in the physics of frustrated or disordered materials [1, 2, 3, 4, 5]. This is because, despite the high heterogeneity of the individuals, and their interactions, not all details are relevant for the emergence of collective patterns. Collective behaviors make social systems interesting for the physicist and, reciprocally, the physicist might contribute with a new perspective to the comprehension of social phenomena.

One of the basic ingredients to be taken into account for modeling people's interactions is imitation, or social contagion. In fact, imitation is observed in diverse social contexts, such as in the dynamics of language learning or decision making. The recurrent conformity to the attitudes, opinions or decisions of other individuals, or groups of individuals, has led to the formulation of models based on social contagion as the primary rule of opinion dynamics, e.g., the voter [6], Sznajd [7] and majority rule [8] models, to give just a few examples. However, aside from imitation, individuals also dissent and resist to be influenced, in several ways.

In the present work, we study the effect of nonconformity attitudes through a kinetic exchange model [3, 9, 10, 11]. Within this modeling, the influences that an individual exerts over another are modulated by a coupling strength. The coupling strength between connected individuals typically takes positive values but also, with certain probability, it can adopt negative ones. Negative values represent negative influences that, instead of imitation, promote dissent. Notice that this kind of dissent is not a characteristic of the individual but of the tie (or link) between each pair of individuals.

There are also other types of nonconformity, which are not associated to the links but to the individuals (or nodes in the network of contacts), as taken into account in several models [12, 13, 14, 15, 16, 17, 18, 19]. One of these types is anticonformity [20, 21]. An anticonformist actively dissents from other people's opinions, which is the case contemplated, for example, by Galam's contrarians [2, 4]. Actually, although these anticonformists defy other people's opinions or the group norm, they are similar to conformists in the sense that they take into account other's opinions too. A different kind of nonconformity is independence, such that the individual tends to resist the influences of other agents or groups of agents, ignoring their choices in the adoption process. It can be thought either as an attribute of the agent, that acts as independent with certain probability, or an attitude that any individual can assume with certain frequency. It is in the latter sense that we will incorporate independence into the model.

Another important ingredient for modeling opinion dynamics is conviction or persuasion (a kind of stubbornness or resistance to change mind) [2, 3, 9, 22, 23, 24, 25, 26, 27, 28, 29]. We will consider the heterogeneity of the individuals in that respect.

Each agent will be characterized by a parameter that measures its level of conviction about the opinion it supports. This parameter, typically defined positive, will be allowed to take also negative values to represent volatile individuals that change mind easily.

Heterogeneities and disorder can act on opinion dynamics as stochastic drivings able to promote a phase transition, playing the role of a source of randomness or noise similar to a social temperature [13, 15, 16, 17, 23, 30]. We will focus on the impact of all the abovementioned sources of disorder on the steady state distribution of opinions in the population, and investigate the occurrence of nonequilibrium phase transitions.

2. Modeling

Our model belongs to the class of kinetic exchange opinions models [9, 10, 11]. We consider a fully-connected population of size N participating in a public debate. Each agent i in this artificial society has an opinion o_i . Most models of opinion dynamics deal with a discrete state space [1, 4, 5], which may be enough to tackle certain problems that involve binary or several particular choices. However, to investigate, for instance, the emergence of extreme opinions, it seems to be more suitable to represent opinions by means of a continuous variable, to reflect the possible shades of peoples' attitudes about a given subject [22, 28, 29, 31, 32, 33, 34, 35, 36, 37]. Then, we will consider a continuous state space, where opinions can take values in the real range $[-1, 1]$. Positive (negative) values indicate that the position is favorable (unfavorable) to the topic under discussion. Opinions tending to 1 and -1 indicate extremist individuals. Finally, opinions near 0 mean neutral or undecided agents.

At a given step s , the following microscopic rules control the opinion dynamics:

- (i) We choose two random agents i and j .
- (ii) With probability q , the agent i acts as independent. In this case, the agent chooses a position about the subject under discussion, i.e., $o_i(s+1)$ is randomly selected from a uniform distribution $[-1, 1]$, independently of the current states $o_i(s)$ and $o_j(s)$. That is, as commented above, we consider that independence is not an attribute of the individual but an attitude that any individual can occasionally assume with certain probability. In that opportunity, individuals choose their own position (state) independently of the other individuals [14, 15, 16, 17].
- (iii) Otherwise, with probability $1 - q$, the agent i acts as a partially conformist individual, and will be influenced by agent j by means of a kinetic exchange. In this case, the opinion of agent i will be updated according to the rule [10, 11]

$$o_i(s+1) = c_i o_i(s) + \mu_{ij} o_j(s) , \quad (1)$$

where c_i and μ_{ij} are real parameters that measure the level of conviction of agent i , and the strength of the influence that agent i suffers from agent j , respectively. The opinions are restricted to the range $[-1, 1]$. Therefore, whenever Eq. (1) yields $o_i > 1$ ($o_i < -1$), it will, actually, lead the opinion to the extreme $o_i = 1$ ($o_i = -1$), as considered in Refs. [9, 10, 11]. This re-injection of the opinions

into the interval $[-1, 1]$ introduces a nonlinearity in the mapping given by Eq. (1), that becomes linear by parts. Smoothing this nonlinearity, for instance through $o_i(s+1) \rightarrow \tanh[o_i(s+1)]$, is not expected to affect the results significantly, as will be discussed below in connection with the outcomes of simulations.

Typically, the strength μ_{ij} is positive, meaning agreement, but it can also take negative values to represent ties that induce dissent. The fraction of negative influences is given by a parameter p , such that μ_{ij} is uniformly distributed either in $[-1, 0]$ or in $[0, 1]$, with probabilities p and $1 - p$, respectively.

Parameter c_i is also typically positive, representing the weight or contribution that the current opinion of agent i has, with respect to that of other agents, in the opinion formation process. The larger c_i , the less other individuals contribute to mold the opinion of agent i . Sufficiently large values of c_i (larger than 1 for the current choice of the set μ_{ij}) mimic intransigence or stubbornness, such that the opinion of agent i will not evolve. Parameter c_i can also take negative values, to represent volatile individuals that are not persistent in their positions, that is, have a propensity to change mind. This spontaneous change must be distinguished from the change of opinion in independent behavior described above. Due to the minus sign, an agent with negative conviction will always change side, adopting a new opinion which is the opposite of the previous one. Differently, in the independent case, since the new opinion is chosen at random, the individual opinion will not necessarily change sign but can take any value in the spectrum.

The fraction of negative convictions will be controlled by a parameter w , such that c_i is uniformly distributed either in $[-1, 0]$ or in $[0, 1]$, with probabilities w and $1 - w$, respectively.

The unit of time is defined by the application of the above-mentioned steps N times. Concerning the random nature of the convictions c_i and the couplings μ_{ij} , which are assumed to be uncorrelated, we consider quenched (frozen) variables, as far as opinion formation supposedly occurs in a time scale much faster than the changes in agents' attributes and couplings.

We will consider populations of N fully-connected individuals, situation which corresponds to a mean-field limit. The initial state of the system is assumed to be fully disordered, that is, at the beginning of the dynamics, each individual has an opinion drawn from the uniform distribution in the range $[-1, 1]$.

In the following, we will describe separately two distinct cases characterized by: (i) the existence of competitive positive/negative interactions among pairs of agents, while convictions are homogeneous (with $c_i = 1$, $\forall i$), and (ii) the heterogeneity of agents' convictions, which interactions are always positive ($\mu_{ij} > 0$, for all i, j). In both cases, the noise introduced by independent attitudes is included.

3. Methods

We calculate the parameter O given by

$$O = \left\langle \frac{1}{N} \left| \sum_{i=1}^N o_i \right| \right\rangle, \quad (2)$$

where $\langle \dots \rangle$ denotes average over disorder or configurations, computed at the steady states. Notice that O is a kind of order parameter that plays the role of the “magnetization per spin” in magnetic systems. It is sensitive to the unbalance between positive and negative opinions. A state with a large value of O ($O \simeq 1$) means that an extremist position reached consensus. Intermediate values indicate: (i) the dominance of either one of the extreme opinions, (ii) that opinions are moderate but one of the sides wins, or (iii) a combination of both. All these states can be identified with ordered ones in the sense that the debate has a clear result (favorable or unfavorable), be extremist or moderate. A small value ($O \simeq 0$) indicates a symmetric distribution of opinions: (i) polarization such that opposite opinions balance, (ii) the dominance of very moderate or undecided opinions around the neutral state, or (iii) a combination of both. In all symmetric cases, the debate will not have a clear winner position. In that sense the collective state can be identified with a disordered one.

We consider also the fluctuations V (or “susceptibility”) of parameter O ,

$$V = N (\langle O^2 \rangle - \langle O \rangle^2), \quad (3)$$

and the Binder cumulant U [38], defined as

$$U = 1 - \frac{\langle O^4 \rangle}{3 \langle O^2 \rangle^2}. \quad (4)$$

All these quantities will be used to characterize the phase transitions between ordered and disordered phases. Additionally, those phases will be described by means of the pattern that the distribution of opinions presents.

4. Results: Competitive interactions and independence

Let us focus first on the effect of competitive interactions (with a fraction p of negative couplings) and independence (which occurs with probability $q > 0$), by studying the case of homogeneous agents with $c_i = 1$ for all i . In Figure 1 we exhibit O as a function of the independence parameter q , for different values of p . As observed in the figure, independence makes the system undergo a phase transition, an effect which is typical of social temperatures [13, 14].

The transition occurs even when couplings are all positive ($p = 0$), then the mere presence of independence leads the system to undergo a phase transition. A similar effect was observed in the discrete version of the present model, considering only opinions $o = -1, 0, 1$, although the critical point $q_c(p = 0)$ has a different value ($1/4$ in the discrete case [17]).

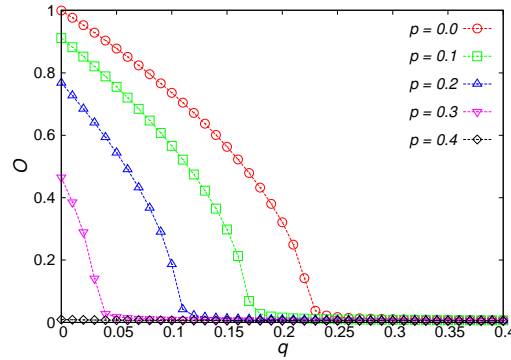


Figure 1. Order parameter O versus the independence probability q , for several values of the fraction p of negative interaction strengths, with $c_i = 1$ for all i . One can observe transitions at different critical points $q_c(p)$, but the maximal value of O decreases with p and the transition is eliminated for sufficiently large values of p . The population size is $N = 10^4$ and data are averaged over 100 simulations.

When we introduce negative interaction strengths ($p > 0$), states characterized by $O = 1$, meaning consensus of extreme opinions, become unlikely (see Figure 1). Moreover, the critical values q_c decrease with p and, above sufficiently large p , the system becomes disordered for any q . The critical value $p_c(q = 0) \approx 0.34$ is in accord with that found in Ref. [11].

To estimate the critical points $q_c(p)$, we performed a finite-size scaling (FSS) analysis. As a typical example, we exhibit in Figure 2 the results of FSS for $p = 0.1$. The critical exponents are $\beta \approx 1/2$, $\gamma \approx 1$ and $\nu \approx 2$. Thus, as expected, the universality class known for the mean-field implementation of the model when $q = 0$ [11] is not altered by the presence of the microscopic disorder introduced by independence.

The FSS analysis also provides the critical values $q_c(p)$, that allow to build a phase diagram in the plane q versus p , depicted in Figure 3. The boundary separates the ordered and disordered phases. In the ordered phase, for $q < q_c(p)$, one of the sides (positive or negative opinions) will win the debate, while in the disordered phase, there will be balance of opposite opinions and/or dominance of moderate ones.

Following the analytical expression found for the discrete kinetic exchange opinion models [23], namely a quotient of two first order polynomials in p , we propose a qualitative description of the phase boundary through

$$q_c(p) = \frac{ap + b}{cp + 1}, \quad (5)$$

which for fitting parameters $(a, b, c) \simeq (-0.667, 0.227, -0.671)$ gives a heuristic description of the phase boundary, in good agreement with the critical points obtained from FSS analysis, as shown in Figure 3. The extreme critical values $q_c(p = 0) \approx 0.227$ and $p_c(q = 0) \approx 0.341$ are given respectively by b and $-b/a$. The latter value is in agreement with the estimate found in Ref. [11], which deals with such limiting case.

Since there is ambiguity in the interpretation of the values of parameter O , we

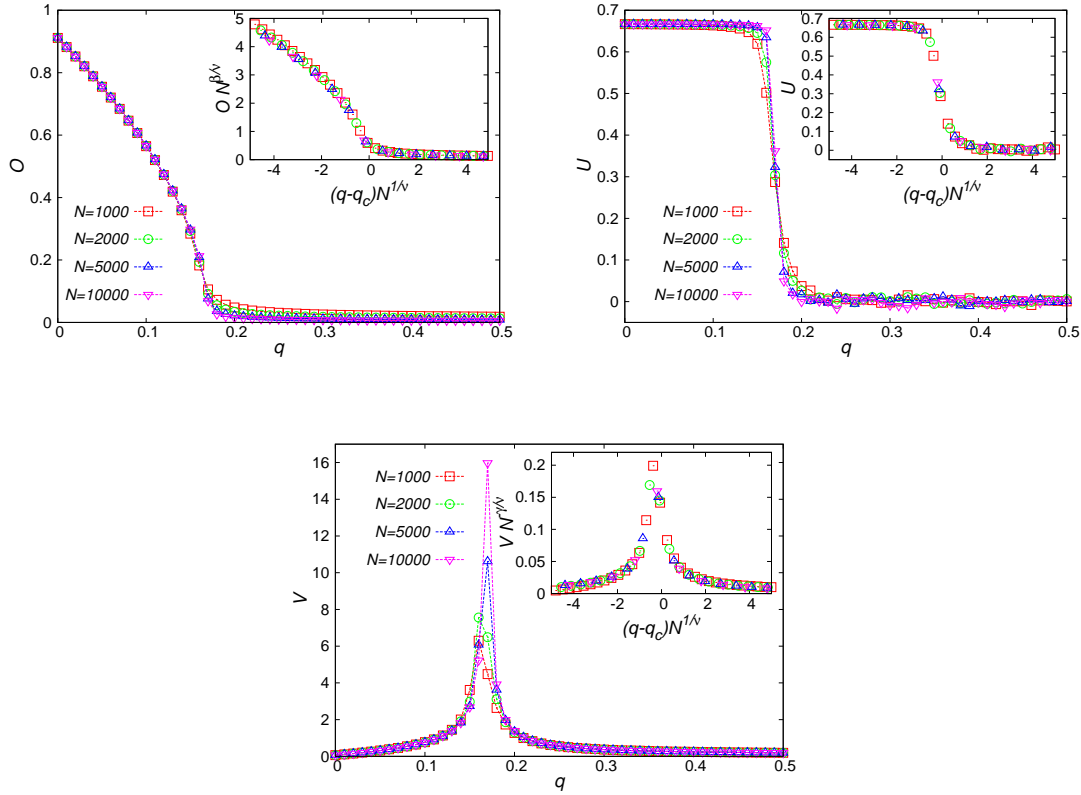


Figure 2. Finite-size scaling analysis of the transition as a function of q , for $p = 0.1$ and $c_i = 1$ for all i . The best data collapse was found for $q_c \approx 0.173$, $\beta \approx 1/2$, $\gamma \approx 1$ and $\nu \approx 2$.

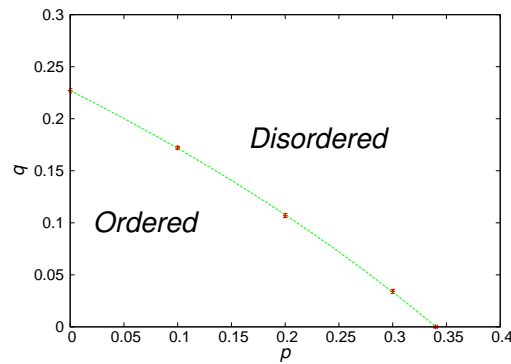


Figure 3. Phase diagram in the plane q (probability of independence) versus p (probability of negative interactions), when $c_i = 1, \forall i$. The symbols are the numerical estimates of the transition points, whereas the dashed line is given by Eq. (5). The error bars were estimated from the FSS analysis.

complemented the previous analysis, computing the distribution of opinions in the population at the steady states of the model. In Figure 4, we exhibit the outcomes for $q = 0$, $q = 0.05$ and $q = 0.2$, at several values of p . Each normalized histogram is

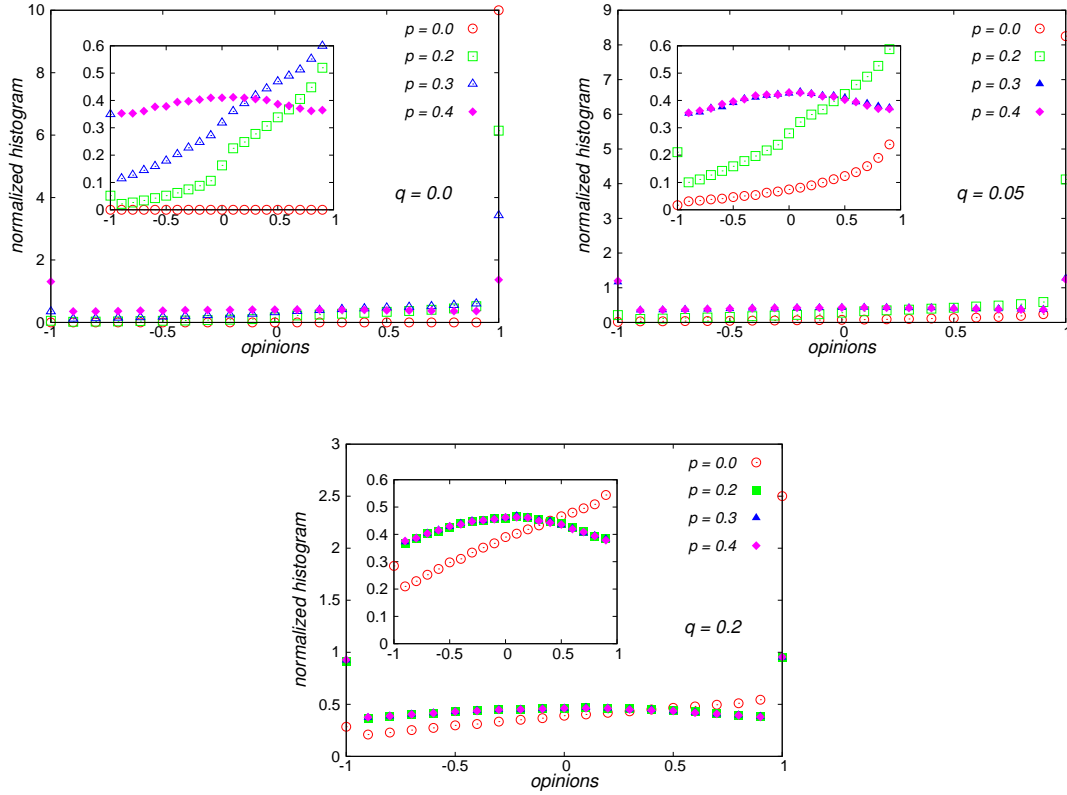


Figure 4. Normalized histograms of opinions, at the steady state, for $q = 0.0$ (top, left panel), $q = 0.05$ (top, right panel) and $q = 0.2$ (bottom), for several values of p , when $c_i = 1, \forall i$. Empty (full) symbols represent ordered (disordered) states. In the insets we show a zoom of the main frames, excluding the extremist agents with majority opinions. The population size is $N = 10^4$, and data are accumulated over 100 independent simulations, as explained in the text.

obtained from 100 independent simulations, for population size $N = 10^4$. When there is unbalance of positive and negative opinions, we arbitrarily selected simulations with dominance of positive opinions to build the histograms. In this way, each histogram is representative of each single realization but with improved statistics. Instead, if we would have chosen the simulations with predominantly negative outcomes, the distribution would be the symmetric counterpart of those shown in Figure 4. First, we notice that there is condensation of opinions at the extreme values $|o| = 1$. This is a consequence of the type of nonlinearity introduced in Eq. (1). If a smoother form were used instead, then there will be a spread of the extreme values in the condensates, which will not affect significantly the number of individuals that can be identified as those who are extremists.

For each frame (fixed value of q), we observe the following scenario. Both the winner and loser sides present a bulk of moderate individuals as well as a condensation at $o = \pm 1$ (extremists). When p is small enough so that the system becomes ordered, unbalance between positive and negative opinions emerges, consistently with a non null

order parameter. When p becomes too large, it is not possible the formation of an ordered phase, where one of the sides dominates, then the distribution becomes flat, with the coexistence of all opinions (except for the symmetrical condensation in the extreme states $|o| = 1$), as can be seen in the insets of Figure 4.

Moreover, we see that the number of extremists (opinions $|o| = 1$) decreases with increasing values of p , as well as with increasing q . That is, the inclusion of dissent interactions, as well as the growth of independent attitudes, inhibit extremism growth, leading to a more homogeneous distribution of opinions, which is associated to the disordered phase in the diagram of Figure 4.

5. Results: Heterogeneous convictions and independence

In this section we consider independence together with the heterogeneity of the convictions c_i , which can take negative values. Recall that we assume that c_i are (quenched) random variables that are uniformly distributed in $[-1, 0]$ with probability w and uniformly distributed in $[0, 1]$ with the complementary probability $1 - w$. Hence, the parameter w denotes the fraction of negative convictions. Positive convictions mean that the agent is sure about the adopted belief and therefore contributes in Eq. (1) to maintain the current opinion. A conviction close to zero reflects uncertainty about the belief and high susceptibility to other people's opinions (conformist behavior), while a negative conviction contributes to form an indifferent or an opposite position to the current one.

When all the couplings are $\mu_{ij} = 1$, the curves for the order parameter O vs q , are qualitatively similar to those shown in Figure 1 (after the substitution $p \rightarrow w$), although the critical values are not the same. Moreover the distribution of opinions are also similar. Hence the cases (i) $c_i = 1$ for all i , and variable p , and (ii) $\mu_{ij} = 1$ for all i, j with variable w , are in some way equivalent. Since case (ii) does not present a new phenomenology with respect to case (i), then we will exhibit, instead of $\mu_{ij} = 1$, the results for a more realistic case, where all the interactions are positive but random, uniformly distributed in the range $[0, 1]$, which corresponds to $p = 0$.

In Figure 5 we exhibit the behavior of the order parameter as a function of q for several values of w . Although the curves are different from those in Figure 1, the effect of convictions is rather similar to that introduced by negative interactions, in the sense that the increase of w , as well as the increase of p , both tend to disorder the system. However, the population does not reach the state $O = 1$ anymore, differently to the case $p = 0$ of Figure 1.

In addition, one observes that the critical values $q_c(w)$ are much smaller than in Sec. 4, where $c_i = 1$ for all i but negative pairwise interactions μ_{ij} (i.e., $p > 0$) were allowed. We estimated the values of the critical points $q_c(w)$ by means of the crossing of the Binder cumulant curves, as well as the critical exponents, as illustrated in Figure 6 for the case $w = 0.2$. These exponents are the same as the ones of the previous section, namely $\beta \approx 1/2$, $\gamma \approx 1$ and $\nu \approx 2$. Thus, the universality class of the model is not

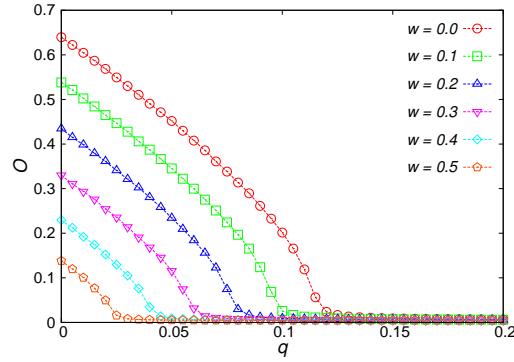


Figure 5. Order parameter O versus the independence probability q , for several values of the fraction w of negative convictions, when $p = 0$. One can observe order-disorder transitions at different points $q_c(w)$, but the transition is suppressed for sufficiently large values of w . The population size is $N = 10^4$ and data are averaged over 100 simulations.

changed by the presence of disorder in the convictions, as expected.

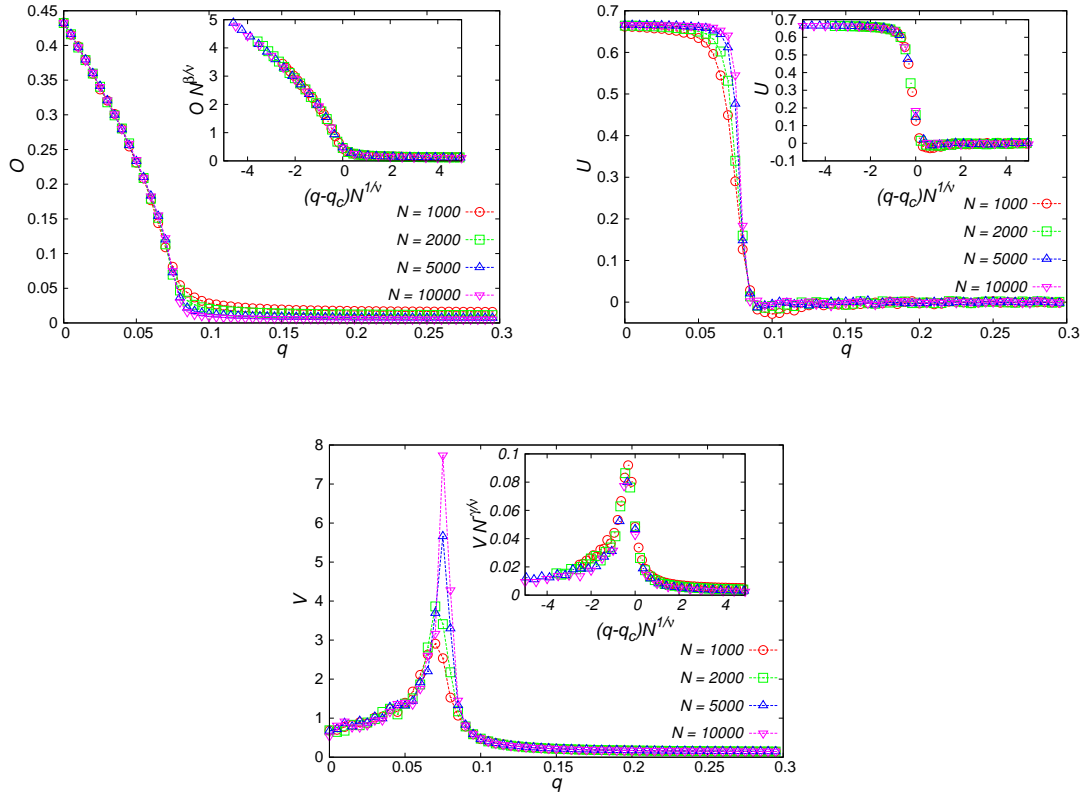


Figure 6. Finite-size scaling analysis of the transition for $w = 0.2$ and $p = 0.0$. The best data collapse was found for $q_c \approx 0.079$, $\beta \approx 1/2$, $\gamma \approx 1$ and $\nu \approx 2$.

Figure 7 shows the resulting phase diagram in the plane q versus w , using the values

of $q_c(w)$ obtained from FSS. The boundary separates the ordered and disordered phases. Following the same analysis of Sec. 4, we estimated the qualitative behavior of the order-disorder frontier by means of Eq. (5), with the change $p \rightarrow w$. The result is plotted in Figure 7 together with the critical points obtained from FSS analysis. One can see a good agreement between the data and the qualitative frontier. In this case, $c \approx 0$ and the extreme critical values are $q_c(w = 0) = b \approx 0.12$ and $w_c(q = 0) = -b/a \approx 0.63$.

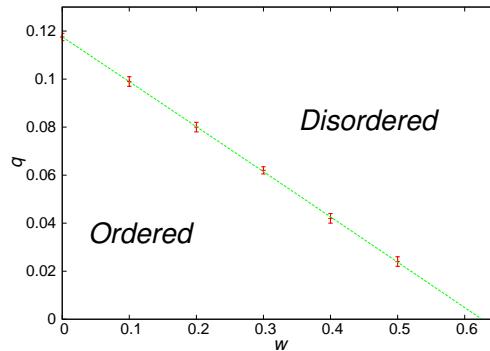


Figure 7. Phase diagram of the model in the plane q (probability of independence) versus w (probability of negative convictions), when $p = 0$. The symbols are the numerical estimates of the transition points, whereas the dashed line is obtained by substituting p by w in Eq. (5), as explained in the text. The error bars were estimated from the FSS analysis.

To understand the nature of the disordered phase, also in this case we obtained the distribution of opinions in the steady state as done in Sec. 4. In Figure 8 we show the normalized histograms for several values of q and w . Each histogram is built from 100 independent simulations, for population size $N = 10^4$, as in Sec. 4. Notice that the distributions are completely different from those in Figure 4, both when the system is in the ordered and disordered phases. For values of the parameters in the disordered phase, i.e. at the right of critical line in Figure 7, the plot is not flat in the center, as it was in the case of Figure 4, but the distribution becomes symmetrically concentrated around the neutral state $o = 0$ where there is a peak. That is, the population becomes essentially neutral and opposite opinions are balanced. When we move away to the left of the critical line in Figure 8, the distribution of opinions loses symmetry, in such a way that one of the sides of the debate (the negative opinions in the example of the figure) tends to disappear, while the opposite (positive) side tends to become uniform, except for the condensation peak at $o = 1$, that is the distribution becomes bimodal. This effect is more pronounced in the absence of negative convictions ($w = 0.0$), which represents the farther points from the frontier, at fixed q .

For increasing values of w , the fraction of extremists decreases (this effect was quantified in Figure 9), as well as the fraction of agents with moderate opinions, while negative opinions emerge in the population.

From other viewpoint, the absence of negative convictions ($w = 0$) leads to the

emergence of extremists in the population, and the introduction of volatile agents with negative convictions makes moderate and indifferent opinions dominant, which seem realistic features of the model. In addition, if the independent behavior becomes more frequent (increasing q), the PDFs become more symmetric and there are less agents sharing extreme opinions. See Figure 9, where we exhibit the fraction of extremists ($|o| = 1$) as a function of w for three different values of q . Notice that the increase of independent attitudes (increasing q) tends to reduce the fraction of extremists in the population. The increase of volatile attitudes (increasing w) favors the emergence of moderate opinions and also reduces condensation at the extreme positions. Indeed, the increase of volatile individuals reduces, even extinguishes, the fraction of extremists and also prevents the arrival to consensus.

Finally, let us comment that when both μ_{ij} and c_i are allowed to take negative values i.e., $p, w > 0$, the phenomenology is similar to that discussed in this section, although the domain of the ordered phase shrinks (not shown, as far as there are not new qualitative features).

Therefore, the opinion patterns observed in this section can be attributed to the joint heterogeneity in the convictions and in the interactions, be them all positive or not. When individuals have all the same conviction $c_i = 1$, or the same interaction strength $\mu_{ij} = 1$, then, patterns of the type observed in Figure 4 emerge. The substitution of $c_i = 1$ by positive heterogeneous convictions, like in the case $w = 0$, is enough to yield patterns of opinions similar to those shown in Figure 8, when we vary p instead of w .

6. Final Remarks

In this work, we have studied a kinetic model of opinion formation with continuous states. We considered pairwise interactions among randomly chosen agents, together with the possibility of independent behaviors that occur with probability q . Two sources of heterogeneity were included: random competitive couplings (interaction term) and random convictions (self-interaction term), that can take negative values with probabilities p and w , respectively. We characterized the critical transitions, as well as the emerging phases, in terms of moderate and extremist individuals.

Parameters p , w and q rule microscopic disorder in the system, and their increase triggers transitions from collective order to disorder. These transitions are characterized by mean-field exponents and their critical curves are shown in Figures 3 and 7.

When only one source of heterogeneity is present, like in the case of Sec. 4, where the convictions are given by $c_i = 1$ for all i , but there is a probability $p > 0$ of negative couplings, patterns of opinions like those shown in Figure 4 occur. Qualitatively similar patterns arise when the pairwise interactions are $\mu_{ij} = 1$ for all i, j but convictions are random. In the disordered phase, opinions are evenly distributed. In the ordered ones, extremists dominate. The steady states show the presence of a large number of extremists with opinions either $o = 1$ or $o = -1$. The fraction of these extremists decreases for increasing values of the independence parameter q , and the system tends

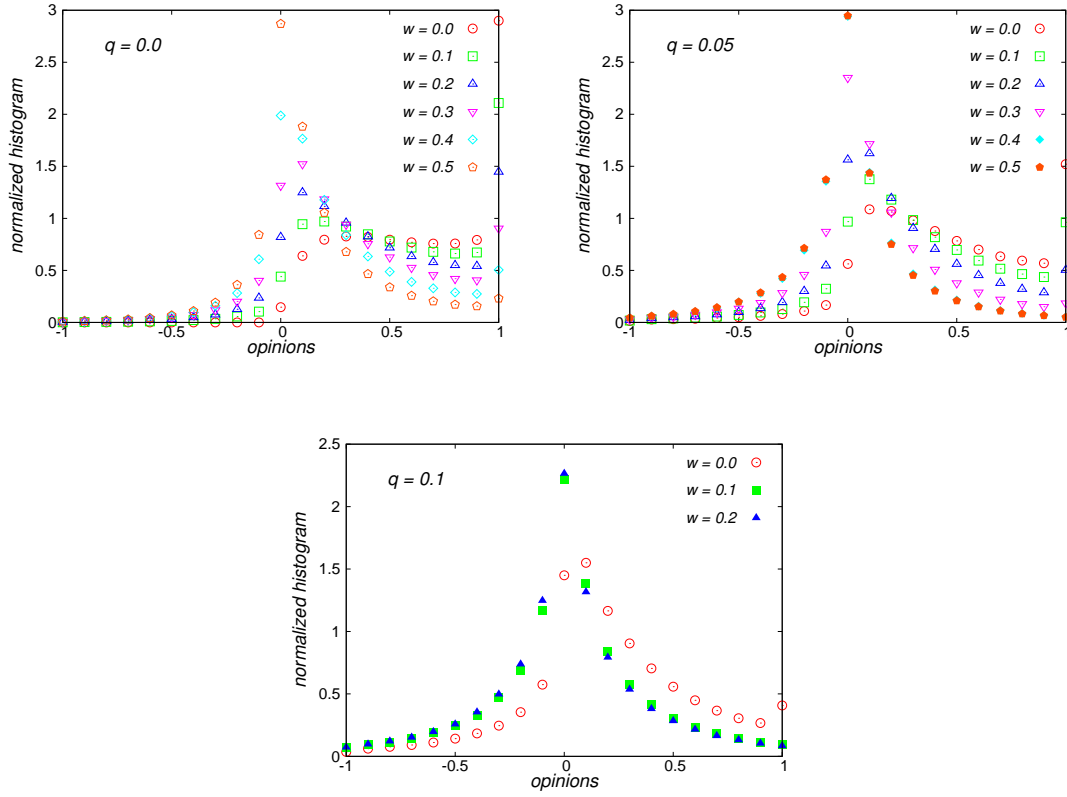


Figure 8. Normalized histograms of opinions, taken at the steady state, for $q = 0.0$ (top, left panel), $q = 0.05$ (top, right panel) and $q = 0.1$ (bottom), for several values of w (and $p = 0$). Empty (full) symbols represent ordered (disordered) states. The population size is $N = 10^4$, and data are accumulated over 100 independent simulations, as explained in the text.

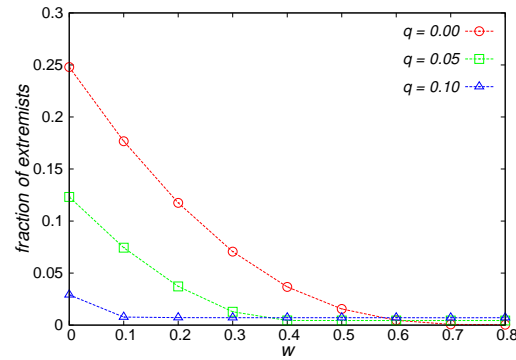


Figure 9. Average fraction of agents with extreme opinions $|o| = 1$ as a function of w (probability of negative convictions) for typical values of q (probability of independence), when $p = 0$. One can see that the number of extremists in the population can be reduced (even to zero) for sufficiently large values of w . Population size is $N = 10^4$ and averages were performed over 100 simulations.

to become disordered, indicating that independent attitudes reduce extreme opinions.

Different patterns emerge when there are two sources of heterogeneity, with negative values or not, like in the case of Sec. 5, where the couplings and the convictions are random. In the disordered phase, opinions are not evenly distributed, but moderate opinions dominate, with a peak around the neutral state. In the ordered phases, one of the opinion sides disappears. When a second source of heterogeneity is present, the increase of volatile individuals (increasing w) or of dissent interactions (positive p), reduces, even extinguishes, the fraction of extremists and also prevents the arrival to consensus.

The models considered in this work contribute to the analysis of the role of the diverse features that characterize individuals and their interactions. As can be seen in this paper, each modification yields phase diagrams of order parameters that seem similar, however a quite different phenomenology arises in terms of the distribution of opinions. This hinders unification, justifying a separate analysis of different versions of the model. These versions shed light on the origin and role of undecided individuals and extremism uprise, and the effect of realistic characteristics like conviction and independence in the dynamics of opinion formation and evolution. In particular, the models contribute to understand the circumstances which favor the emergence and development of extremism in a fraction of the population, relating that emergence with the existence of individuals with strong positive convictions in the population. On the other hand, the presence of negative convictions, that represent volatile individuals that have a propensity to change mind, as well as nonconformity represented in the model by the independent behavior, lead to the dominance of moderate individuals.

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